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Stronger correlations than dense coding with the same quantum resources

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Dense coding is the seminal example of how entanglement can boost quantum communication. By sharing an Einstein-Podolsky-Rosen (EPR) pair, dense coding allows one to transmit two bits of classical information while sending only a single qubit [1]. This doubling of the channel capacity is the largest allowed in quantum theory [2]. In this letter we show in both theory and experiment that same elementary resources, namely a shared EPR pair and qubit communication, are strictly more powerful than two classical bits in more general communication tasks. In contrast to dense coding experiments [3–8], we show that these advantages can be revealed using merely standard optical Bell state analysers [9, 10]. Our results reveal that the power of entanglement in enhancing quantum communications qualitatively goes beyond boosting channel capacities.

Entanglement and quantum communication are both paradigmatic resources for quantum information science and crucial for understanding the nonclassical nature of quantum theory. The former has been studied for decades in Bell-type experiments [11–14] where communication is absent. The latter has, in more recent years, been extensively studied in prepare-and-measure experiments, where shared entanglement is absent [15–17]. It is natural to investigate the most general scenario, namely when entanglement and quantum communication are combined.

The power of entanglement in quantum communication experiments was originally, and most strikingly, illustrated by dense coding (Fig. 1a). A sender and receiver initially share an EPR pair $|\phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$. Operating only on her local qubit, the sender can create a global basis of four orthogonal, maximally entangled states. Therefore, when the sender's qubit is relayed to the receiver, two bits of information can be retrieved. This sharply contrasts scenarios without entanglement, in which a qubit can never carry more than one bit of information [18]. In the years following, substantial effort was directed at understanding how, and to what extent, entanglement can be used to transmit information over a quantum channel [2, 19, 20].

The general predictions of quantum theory, however, are far richer than those associated to the task of reliably transmitting classical information over a quantum channel. For example, although qubits and bits carry the same amount of information, there exist scenarios, without entanglement, in which the former enables correlations that are impossible with

FIG. 1: (a) *Dense coding scenario.* The sender (S) and receiver (R) share an EPR pair $|\phi^+\rangle$. The sender selects one of four Pauli unitaries $\{\mathbb{I}, \sigma_X, \sigma_Y, \sigma_Z\}$, applies it to her qubit and relays it to the receiver who performs a measurement of both qubits in the basis of the four Bell states $|\phi^\pm\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}$ and $|\psi^\pm\rangle = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}$. From the outcome, the receiver's two bit input can be perfectly recovered. (b) *Our communication scenario.* The parties again share an EPR pair and are allowed to communicate a qubit. However, the receiver can now ask several binary questions about the sender's input.

the latter [21, 22]. This requires communication tasks that are more general than perfect information recovery. However, when both entanglement and quantum communication are available, little is known about such tasks. In view of this, it is important to understand the role of the dense coding

protocol in the broader landscape of correlations predicted by quantum theory. We investigate the correlations powered by the same elementary quantum resources used in dense coding. We show that the communication of a single qubit, assisted by a shared EPR state, can give rise to predictions that cannot be reproduced by any dense coding protocol or, equivalently, any classical protocol using two bits of communication. Furthermore, we show that these advantages can be subject to experimental verification.

Consider a communication task in which the sender selects a classical input x and encodes it into a message, which is sent to the receiver. The receiver selects a question, labeled y , to which he produces an answer labeled b . When repeated many times, the experiment produces statistics described by the conditional probability distribution $p(b|x, y)$. Here, we are interested in a setting in which the sender and receiver pre-share the entangled state $|\phi^+\rangle$ and the sender is allowed to send only a single qubit. Equipped with these resources, the parties could implement the dense coding protocol to produce every possible distribution $p(b|x, y)$ when x takes at most four different values, regardless of how many questions the receiver asks. Therefore, to identify quantum correlations that go beyond those attainable by dense coding, we must consider scenarios in which the sender has at least five different inputs and the receiver asks more than two binary questions¹.

Here, we focus on a specific communication task in which the sender selects an input $x \in \{1, \dots, 5\}$ and the receiver selects one of six questions, $y \in \{1, \dots, 6\}$, each with a binary answer $b \in \{+1, -1\}$. This is illustrated in Fig. 1b. Clearly, in contrast to dense coding, the correct answer to any single question can only provide partial knowledge about x . We consider a simple figure of merit in which each question either has precisely one correct answer or no correct answer. Using normalisation, $p(b = +1|x, y) + p(b = -1|x, y) = 1$, we can rephrase this in terms of the answer “ $b = +1$ ” either being awarded one point (if correct), being penalised by one point (if incorrect) or being ignored. The task is to maximise the total number of points:

$$\mathcal{S} \equiv \sum_{x=1}^5 \sum_{y=1}^6 c_{xy} p(b = +1|x, y), \quad (1)$$

where the points awarded for each setting are given by

$$c = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & -1 & -1 & 1 \\ 0 & 0 & -1 & -1 & -1 & -1 \end{pmatrix}. \quad (2)$$

The figure of merit is chosen based on its simplicity and, as it turns out, its favourable theoretical and experimental properties

¹ Otherwise, rather than encoding her message, the sender could send the answer to these questions.

The highest attainable score $\mathcal{S}$ based on a dense coding protocol is equal to the highest score using two classical bits. This can be computed explicitly, $\mathcal{S}_{2 \text{ bits}} = 5$ (see methods). An explicit strategy has the sender use her two bits to encode $\{x = 1, x = 2 \vee 3, x = 4, x = 5\}$. Indeed, a protocol based on dense coding substantially improves on the best standard classical protocol based on one bit of communication (at best $\mathcal{S}_{\text{bit}} = 3$) and even on the best protocol when one bit of communication is assisted by any amount of shared entanglement (at best $\mathcal{S}_{\text{ent+bit}} \approx 3.799$, see methods).

However, there are quantum protocols, based on the same resources, that outperform the best protocol based on dense coding. In such protocols, the sender performs more general unitaries than those used in dense coding on her local share of $|\phi^+\rangle$ and relays this qubit to the receiver, who in turn performs simple dichotomic measurements. Specifically, in one such protocol, the sender applies the following unitary transformations U_x^S on their qubit:

$$U_1^S = \mathbb{1}, \quad U_2^S = \frac{-\sigma_Z\sqrt{3} - \sigma_X}{2}, \quad U_3^S = \frac{\sigma_X\sqrt{3} - \sigma_Z}{2}, \\ U_4^S = \frac{\mathbb{1} - i\sigma_Y\sqrt{3}}{2}, \quad U_5^S = \frac{\mathbb{1} + i\sigma_Y\sqrt{3}}{2}. \quad (3)$$

These unitaries correspond to rotations in the XZ -plane of the Bloch sphere. When the sender's qubit arrives to the receiver, the joint state of the two qubits is $U_x^A \otimes \mathbb{1}|\phi^+\rangle$. Depending on the question y , the receiver performs a binary-outcome measurement $\{|e_y\rangle\langle e_y|, \mathbb{1} - |e_y\rangle\langle e_y|\}$. We let the outcome $b = +1$ correspond to a projection onto the state $|e_y\rangle = U_y^R \otimes \mathbb{1}|\phi^+\rangle$ for some unitary transformation U_y^R . Such a measurement may be viewed as a locally rotated partial Bell state analyser; it attempts to discriminate the maximally entangled state $|e_y\rangle$ from its three-dimensional orthogonal complement. We choose the unitaries of the receiver as

$$U_1^R = \mathbb{1}, \quad U_2^R = \frac{\nu_+ \mathbb{1} + i\nu_- \sigma_Y}{2\sqrt{2}}, \quad U_3^R = \frac{\nu_+ \mathbb{1} - i\nu_- \sigma_Y}{2\sqrt{2}}, \\ U_4^R = U_2^S, \quad U_5^R = U_3^S, \quad U_6^R = \frac{\mathbb{1} - i\sigma_Y}{\sqrt{2}}. \quad (4)$$

where $\nu_{\pm} = \sqrt{3} \pm 1$. Once again, these unitaries correspond to rotations in the XZ -plane. We evaluate our figure of merit using the Born rule,

$$\mathcal{S} = \sum_{x,y} c_{xy} |\langle e_y | U_x^S \otimes \mathbb{1} | \phi^+ \rangle|^2 \\ = \frac{1}{2} \text{tr} \left[(U_y^R)^\dagger U_x^S \right] = 3 + \frac{3\sqrt{3}}{2} \approx 5.598 \quad (5)$$

This protocol is robust to unavoidable implementational imperfections. For instance, if the EPR pair is exposed to isotropic noise, so that the state becomes $v|\phi^+\rangle\langle\phi^+| + \frac{1-v}{4}\mathbb{1}$, for some visibility $v \in [0, 1]$, the advantage over dense coding is maintained whenever $v > \frac{16}{12+3\sqrt{3}} \approx 93\%$. Finally, we note that the value (5) can be somewhat further increased. Using a numerical search, we find a protocol achieving $\mathcal{S} \approx 5.641$. However, this protocol requires more complicated measurements than those based on partial Bell state analysers.

FIG. 2: Ultraviolet light centred at a wavelength of 390 nm is focused onto two 2 mm thick β barium borate (BBO) nonlinear crystals placed in cross-configuration to produce photon pairs emitted into two spatial modes (a) and (b) through the second order degenerate type-I SPDC process. The spatial, spectral and temporal distinguishability between the down-converted photons is carefully removed by coupling to single mode fiber, narrow Filter (F) and quartz wedges respectively. The unitaries of the sender and receiver are implemented using half wave plates (HWP) and phase shifters (PS). The partial Bell state measurements are implemented through two-photon interference, using PBS and HWP plates set at 22.5° . The polarisation measurements are performed by using HWPs and polarising beam splitters (PBS) followed beam splitter (BS) and single photon detectors (actively quenched Si-avalanche photodiodes, DET). Outcome $b = +1$ corresponds to projection onto $|\phi^+\rangle$, and outcome $b = -1$ corresponds to the other Bell states $|\psi^-\rangle$, $|\psi^+\rangle$, and $|\phi^-\rangle$.

The favourable properties of the quantum protocol make it particularly suitable for implementation using the polarisation degree of freedom of single photons. Using a spontaneous parametric down-conversion (SPDC) process, we prepare two-photon polarisation entangled state $|\Psi\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$. The single-qubit unitaries are implemented using combinations of wave-plates and phase shifters while the partial Bell state measurement is implemented by interfering the two photons via a polarising beam splitter. The setup is illustrated in Fig. 2 and the specific settings are given in Supplementary Material (SM). The average two-photon coincidences rate is about 2500 per second. We benchmark the state preparation by measuring an average visibility of 0.992 ± 0.001 in the diagonal polarisation basis. Similarly, we benchmark the two-photon interference by a two-fold Hong-Ou-Mandel dip visibility of 0.961 ± 0.002 . For each setting x and y , we collect on average 18 million events during a measurement time of two hours. The probabilities $p(b|x, y)$ are estimated from the relative frequencies (see SM). This leads us to the experimentally measured value of $\mathcal{S} = 5.379 \pm 0.009$, which outperforms the dense coding limit by approximately 40 standard deviations. Due to the sizeable violation and the large number of collected events, the p -value associated to our falsification of a dense coding based model is vanishingly small. The result and its relation to the various theoretical limits is illustrated in Fig. 3.

Our finding, that entanglement and quantum communication combined allow for more powerful quantum correlations than those associated to the paradigmatic dense coding protocol, is based on departing from the study of reliable information transfer in favour of more general quantum com-

munication tasks. Already in the simplest nontrivial setting, namely that in which the sender selects from five symbols, we find that the elementary quantum resources consumed in dense coding enable stronger correlations. Interestingly, these advantages can be paired with practical advantages: the complete Bell state measurement [1] needed for dense coding, which complicates optical implementations [23–25], can be substituted for a simpler measurement compatible with linear optics without auxiliary degrees of freedom. Experimentally, this paves the way for setup-efficient ways of harvesting quantum advantages in these, more general, communication scenarios. Theoretically, our result motivates a research effort [26, 27] to systematically explore, understand and apply the correlations arising from the conjunction of entanglement and quantum communications.

METHODS

Correlation bounds.— When communication is classical and no entanglement is present, $p(b|x, y)$ can be geometrically represented as a polytope whose vertices correspond to deterministic encoding and decoding schemes [15]. Consequently, the optimal performance of any linear figure of merit, e.g. that in Eq. (1), is necessarily attained at a vertex of this polytope. By checking all such strategies, we find $\mathcal{S}_{\text{bit}} = 3$ and $\mathcal{S}_{2\text{bits}} = 5$. However, when (potentially unbounded) entanglement is added, this picture breaks down. Instead, upper bounds on $\mathcal{S}$ can be determined using the hierarchy of semidefinite programming relaxations developed in [26], which uses the concept of informationally-restricted quantum correlations

FIG. 3: Illustration of the experimentally measured performance of the communication task and its comparison to the best conceivable protocols based on one bit of classical communication, one bit of classical communication assisted by unbounded entanglement and two bits of classical communication. The latter bound is equal to the maximum attainable value using a dense coding protocol. The right-most dashed line represents the theoretical value of the targeted quantum protocol, based on a shared EPR pair and a communicated qubit, which goes beyond the limitations of dense coding.

[28, 29]. Using this method, and matching it with an explicit entanglement-based strategy with classical communication, we find $S_{\text{ent+bit}} \approx 3.799$.

Statistical significance.— To express the statistical significance of our experimental results, we follow an approach similar to [30] introduced by [31], to which we refer for details. Consider the random variable

$$\hat{S}_i = \sum_{xy} c_{xy} \frac{\chi(b_i = 1, x_i = x, y_i = y)}{p(x, y)}, \quad (6)$$

where i corresponds to the i th experimental run, $\chi(e)$ is the indicator function for the event e , i.e. $\chi(e) = 1$ if the event is observed and $\chi(e) = 0$ otherwise. For our experiment we simply chose $p(x, y) = 1/(6 \times 5) = 1/30$. The random

variable $\hat{S}_i$ may depend on past events, $j < i$, but not on future events, $j > i$. We define $\hat{S} = \frac{1}{N} \sum_{i=1}^N \hat{S}_i$ as our estimator for the value of our scoring function S defined in (1), where $N \sim 18 \times 5 \times 6$ million is the total number of experimental rounds.

The Azuma-Hoeffding inequality implies that the probability p that dense coding or, equivalently, a two bit communication model will yield a value of S greater or equal to the observed value is bounded by

$$p\left(\frac{1}{N} \sum_{i=1}^N \hat{S}_i \geq S_{2\text{bits}} + \mu\right) \leq \exp\left(-\frac{2N\mu^2}{(c+T)^2}\right), \quad (7)$$

where $\mu = 0.379$ is the observed violation of the two bit bound, $T = 9$ is the classical 2-bit bound on $-S$ and $c \equiv \max_{xy} c_{xy}/p(x, y)$. One finds that this probability is vanishingly small.

Experimental errors.— To reduce the multi-photons pairs emission we worked at a low rate (≈ 2500 pairs per sec) and increased the measurement time to reduce statistical errors. The impact of systematic errors was estimated using Monte Carlo simulation. These were reduced by using computerised high precision mounts. (See details in SM). The experiment is performed using the fair sampling assumption.

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